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IN-PLANE MOIRE STRAIN ANALYSIS OF BENT PLATES

A. J. DURELLI, K. CHANDRASHEKHARA and V. J. PARKS

The Catholic University of America, Washington, D.C.

Abstract—A simple experimental method to strain analyze bent plates is described which has advantages in many cases with respect to the conventional determination of slopes using the out-of-plane moire method. The new method consists in determining in-plane displacements using in-plane moire techniques. Strain and stress distributions can be readily obtained from the moire patterns for plates subjected to small as well as large deflections. Two examples of plate bending, one with small deflection and the other with large deflection, are included as illustrations. These are respectively: a simply supported square plate under uniform pressure, and a square cantilever plate subjected to corner load.

NOTATION

a	length of the square plate
D	flexural rigidity
E	modulus of elasticity
h	thickness of the plate
n	moire fringe order
P	concentrated load
p	grating pitch
q	lateral pressure per unit area
u, v, w	Cartesian displacement components in the x, y and z direction (w = deflection of plate)
x, y, z	Cartesian coordinates
γ_{xy}	Cartesian shear strain component
ν	Poisson's ratio
ρ	radius of curvature
σ_x, σ_y	bending stresses in x and y directions
τ_{xy}	shear stress

INTRODUCTION

THE well known moire method developed by Lightenberg [1] and later applied by Bradley [2] is widely used for the experimental strain and stress analysis of bent plates. This method requires the manufacturing of a model of the plate made with a reflecting surface which is used as a mirror to observe a coarse grating of equidistant parallel lines. When the model is loaded the image of this grating changes its position due to the change in slope of the bent plate. By superposing the two images, the one obtained before and the one obtained after loading (e.g. by double exposure on the same film), moire fringes are produced by interference. Two such moire patterns obtained with the grating set in orthogonal directions (for example x and y direction) are required for the complete solution of the problem. The two moire patterns so obtained represent contour lines of the slope of the bent plate. The moment distribution in plates can then be determined by differentiation. Since this slope is associated with an out-of-plane displacement, this method may be called an out-of-plane moire method. A brief review of other experimental methods to determine slopes of bent plates may be found in a paper by Duncan and Brown [3].

LIMITATIONS OF OUT-OF-PLANE MOIRE METHOD

The following strain–curvature relationships are the basic equations that apply to plates subjected to lateral loads and small deflections.

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2}, \; \varepsilon_y = -z\frac{\partial^2 w}{\partial y^2}, \; \gamma_{xy} = -2z\frac{\partial^2 w}{\partial x \partial y}. \tag{1}$$

The out-of-plane moire method has distinct limitations when applied to the solution of some bending problems like those present in plates with large deflections, in plates subjected to combined in-plane and lateral loads and in laterally loaded plates with discontinuities, the size of which is of the order of the thickness of the plate. The following are some of the difficulties that may be encountered:

(a) Since the strain–curvature relationship, equation (1), used for the interpretation of out-of-plane moire data is based on the small deflection theory of laterally loaded plates, errors may be introduced when it is applied to plates subjected to large deflections or to combined in-plane and lateral loads.

(b) In case of plates with relatively small cut-outs, the out-of-plane moire pattern may not show the effect of the cut-out since strains near small cut-outs are no longer proportional to the curvature of the plate.

IN-PLANE MOIRE METHOD

Another moire method which differs fundamentally from the one described above has been developed by Dantu [4]. Moire effects obtained using this method are due to the interference of two gratings which usually are very dense and made of equidistant parallel lines: one printed on the specimen that will be deformed and the other on a master used as reference.

Moire fringes obtained by the in-plane moire method are loci of points exhibiting the same value of components of displacements (u or v for instance) in a direction perpendicular to the grating lines. These loci are called isothetics. Two moire patterns obtained with gratings in orthogonal directions (for example, the x and y directions) represent the displacement field in the plane of the specimen. If the moire fringes are numbered consecutively, starting from a point of zero displacement, then the value of the component of displacement for all points at the nth fringe is np where p is the pitch (distance between two consecutive lines of the master grating). This can be written as

$$u, v = np \tag{2}$$

where u and v are the components of displacement in the x and y direction respectively. Since the moire patterns obtained using this method give directly the in-plane displacements, this method may be called an in-plane moire method.

From these moire patterns, the displacements u and v along any line of interest can be plotted and the values of $\partial u/\partial x$, $\partial v/\partial y$, $\partial u/\partial y$ and $\partial v/\partial x$ may be obtained graphically either by drawing tangents to the displacement curve or by obtaining the secants for equal segments on the curve. Using these derivatives, strain distribution in the specimen [5, 6] can be determined from the strain–displacement relationship. It should be pointed out that these strains correspond to the Eulerian description. This method has been used for strain analysis of a large number of two dimensional and three dimensional problems [7–9].

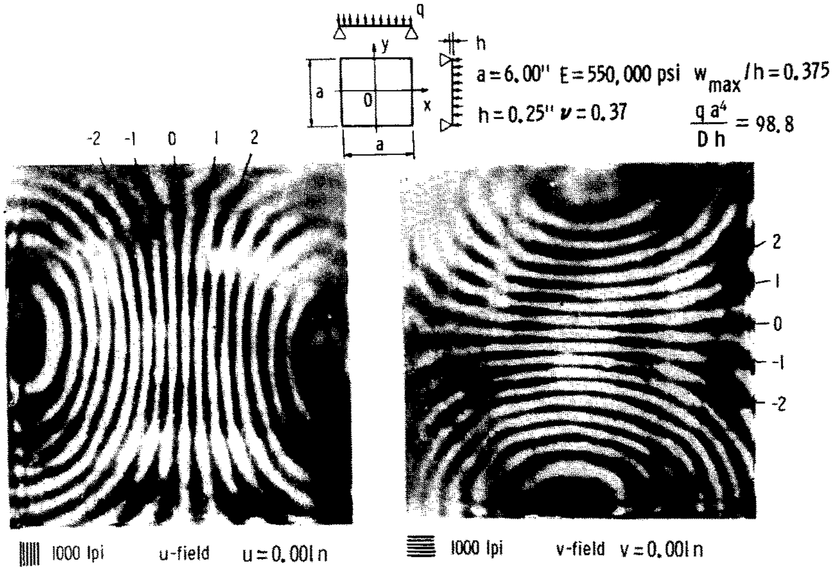

FIG. 1. Isothetics in a simply-supported plate under uniform pressure (small deflection).

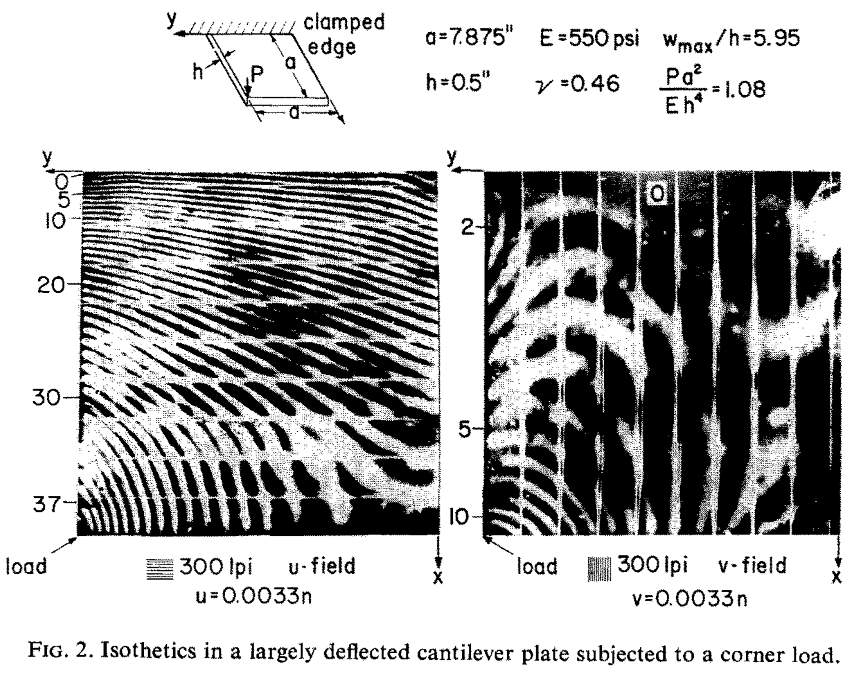

FIG. 2. Isothetics in a largely deflected cantilever plate subjected to a corner load.

The technique used here to obtain the in-plane moire pattern for bent plates is somewhat different from the technique used in the conventional moire method applied to two-dimensional problems.

The moire pattern for the bent plates is obtained by superposing a master grating on the deformed grating of the plate and the master grating follows the bent surface of the plate. By this procedure the master grating is subjected to strain due to bending. However, the error introduced by this strain can be computed. For example, assume that the plate and the master grating film are subjected to cylindrical bending. The strain in the plate and the grating will be of opposite sign and given by

$$\varepsilon = \pm\frac{h}{2\rho} \tag{3}$$

where h is the thickness and ρ is the radius of curvature.

The radius of curvature will be the same for plate and film, so that the unwanted bending strain of the master will be in the same ratio to the bending strain of the plate as the ratio of film thickness to the plate thickness. If the film thickness is of the order of 0·003 in. the plate should be at least 0·100 in. thick for the error to be less than 3 per cent.

To have a perfect contact between the specimen and master gratings during bending of the plate, a thin film of oil is used between them. It is assumed that this thin oil film keeps the master in contact with specimen without transmitting any shears.

The moire pattern, obtained for the bent plate using the above procedure, can be directly photographed using a camera if the deflection of the plate is small. However, if the deflection of the plate is large, this procedure does not give a pattern readily interpretable. To overcome this difficulty a new technique, which does not require the use of a camera and will be described later, has been used.

In this paper the following two cases have been analyzed: (i) a simply supported square plate subjected to a uniformly distributed load producing small deflection; (ii) a square cantilever plate subjected to a concentrated load at one corner, producing large deflections.

A theoretical solution of the stress distribution for the first problem is available [10]. For the second problem, a general solution of the displacements and moments has been given recently by Lin *et al.* [11].

EXPERIMENTAL WORK

(i) *Simply supported square plate under uniform pressure*

A 1000 lines-per-inch grating was printed on the surface of a plexiglas plate $6 \times 6 \times \frac{1}{4}$ in. The plate was placed in loading fixture in such a way that it was simply supported along the four sides leaving a $\frac{1}{4}$ in. overhang all around. A master grating of 1000 lines-per-inch was placed in contact with the printed surface of the plate. A thin layer of paraffin oil was used between the master grating and the specimen to insure contact without producing air gaps. A column of mercury applied the uniform pressure to a sealed cell that rested against the back surface of the plate. The moire patterns produced in the loaded plate were photographed using a camera. Typical u and v moire patterns are shown in Fig. 1.

(ii) *A square cantilever plate subjected to corner load*

The case of a square cantilever plate subjected to a concentrated load at one corner was chosen to illustrate the application of in-plane moire method to laterally loaded plates with large deflections. The size of the plate was 7·875 × 7·875 × 0·5 in. thick and was made from a polyurethane rubber sheet (Hysol 4485). A three hundred lines-per-inch cross grating was printed on the plate. The plate was mounted vertically and clamped along the bottom edge. The load was applied at one corner on the top by means of a string going over a pulley.

A three hundred lines-per-inch one way grating was used for the master, printed on a photographic film 0·003 in. thick. In order to record the undistorted image of the isothetics on the surface of the specimen, a simple photographic technique that does not require the use of a camera [12] was used here. A sheet of photographic film (covering the full size of the specimen) was placed on top of the plate and in contact with the master grating. A diffused white light source on the opposite side of the plate was used to expose the film transmitting the light through the transparent specimen. The photograph obtained by this technique gives a correct moire pattern that can be used for the analysis. A typical pattern obtained from a largely deflected plate is shown in Fig. 2.

In order to verify the results obtained by out-of-plane and in-plane moire methods, a test was also conducted on a square aluminum cantilever plate $9 \times 9 \times \frac{1}{16}$ in. using electrical strain gages. The position of the strain gages are shown in Fig. 7.

ANALYSIS AND RESULTS

Two families of moire patterns, u and v isothetics were recorded for each level of load applied to the plates. From these patterns the displacements u and v along any line of interest were plotted and the values of $\partial u/\partial x$, $\partial v/\partial y$, $\partial u/\partial y$ and $\partial v/\partial x$ obtained graphically. These values were then used in the strain displacement relations to determine the strains. From the strains and stress–strain relationship, stresses were determined.

Figure 3 shows the deflection curve for the simply supported plate obtained by integration, equation (1). The theoretical deflection curve [10] obtained for the same case is also included in this figure for comparison. The strain components ε_x and ε_y along the center (x-axis) of the plate are shown in Fig. 4. Normalized bending stresses σ_x and σ_y along the center of the plate are shown in Fig. 5 together with the results of the theoretical solution.

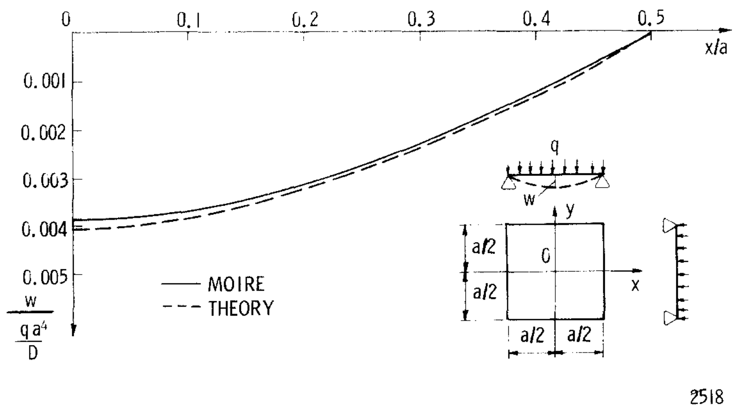

FIG. 3. Deflection curve along the x-axis of a simply supported square plate under uniform pressure.

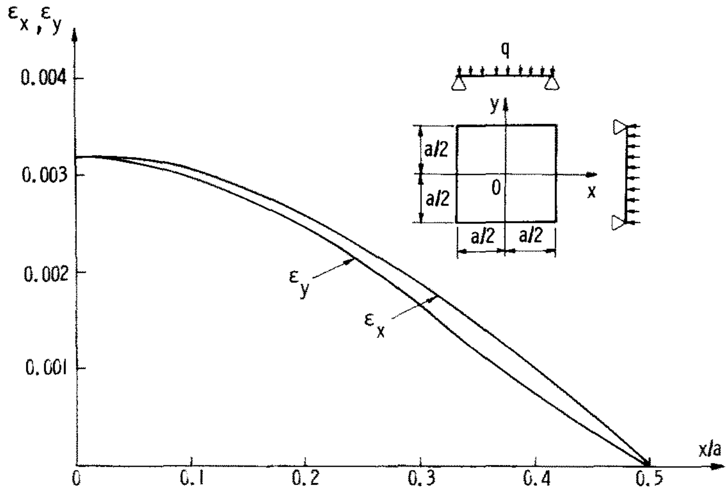

FIG. 4. Cartesian strain components ε_x and ε_y along the x-axis of a simply supported square plate under uniform pressure.

Figure 6 shows the nonlinear relationship between the applied load and (1) the maximum deflection and (2) the displacements at the loaded point, for the cantilever plate subjected to different levels of load. The bending moment coefficient k at different sections in the cantilever plate is shown in Fig. 7 in which the results obtained by out-of-plane moire [13]

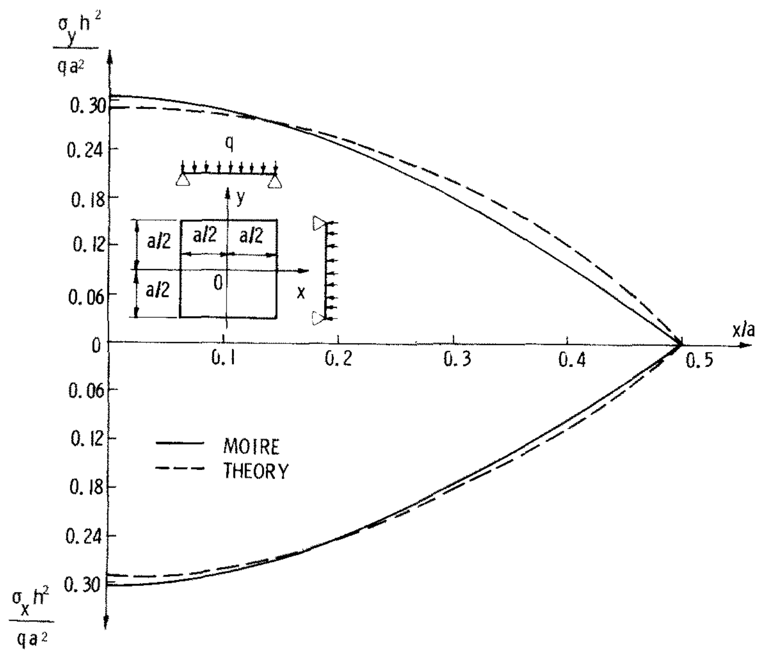

FIG. 5. Bending stresses σ_x and σ_y along the x-axis of a simply supported square plate under uniform pressure.

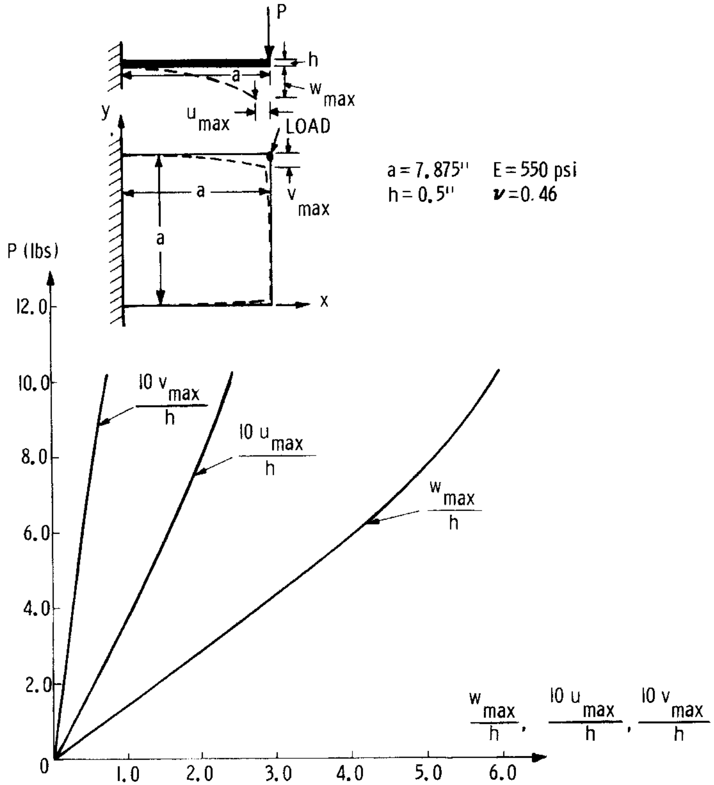

FIG. 6. Maximum deflection and displacements at the loaded point in a cantilever plate subjected to different levels of load at the corner.

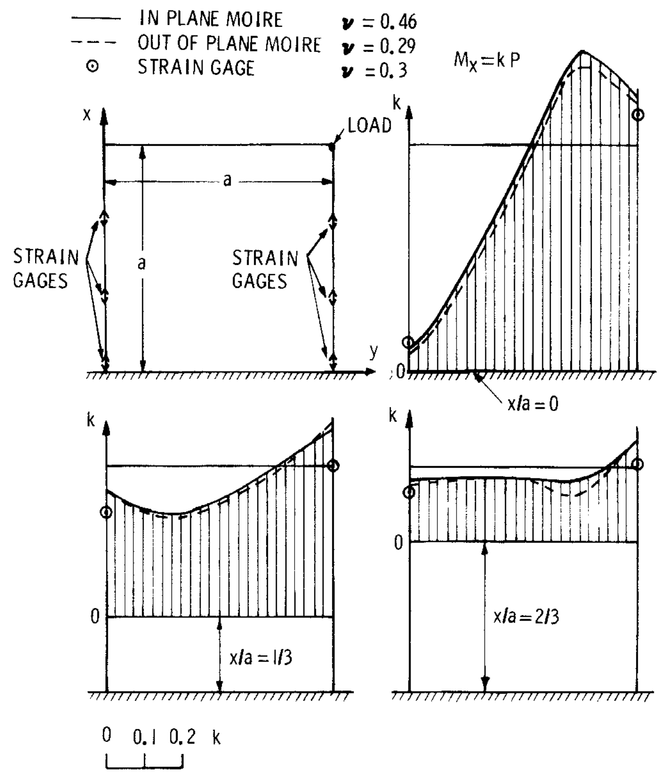

FIG. 7. Bending moment (M_x) at different sections in a square cantilever plate subjected to a corner load.

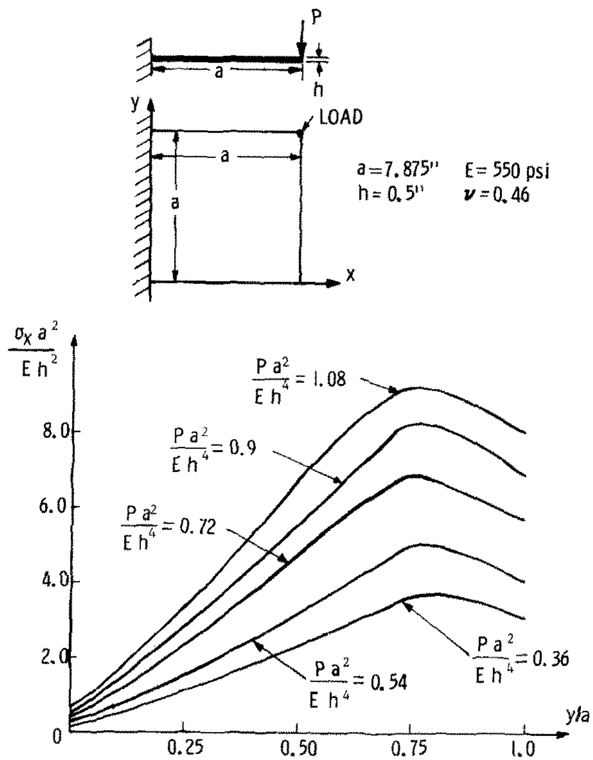

FIG. 8. Bending stress σ_x along the top surface of the clamped edge of a square cantilever plate subjected to different load levels at the corner.

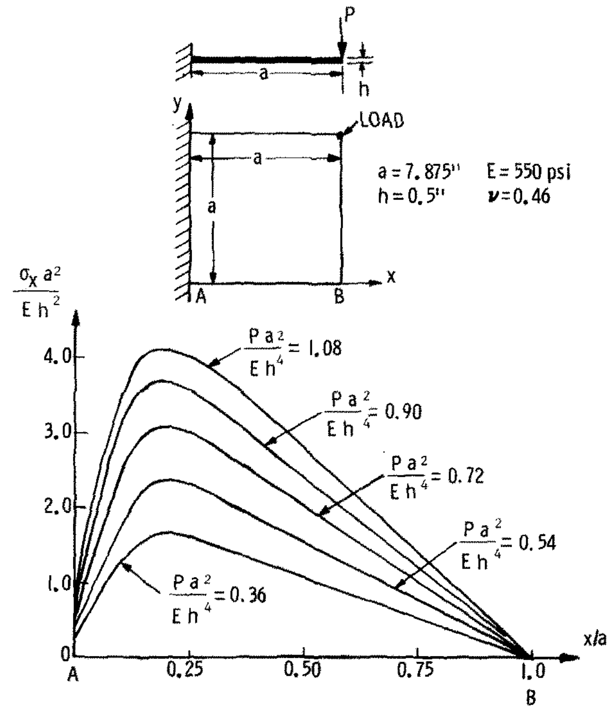

FIG. 9. Bending stress σ_x along the top surface of the edge AB of a square cantilever plate subjected to different load levels at the corner.

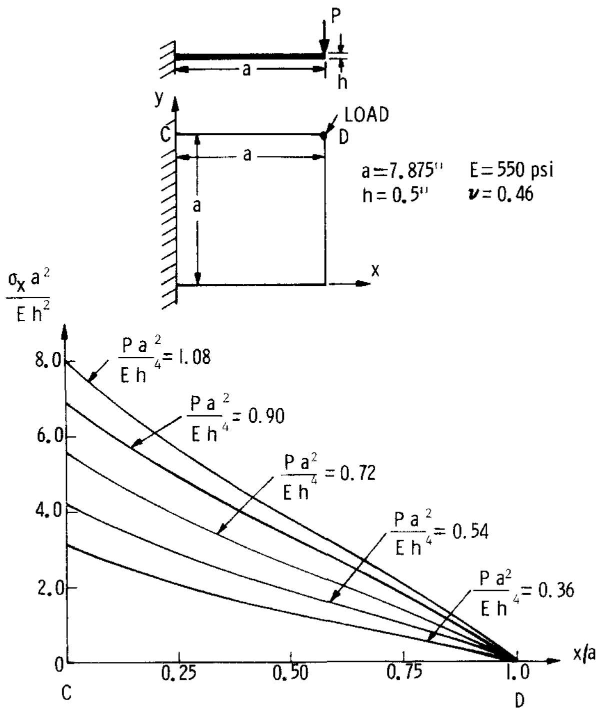

Fig. 10. Bending stress σ_x along the top surface of the edge CD of a square cantilever plate subjected to different load levels at the corner.

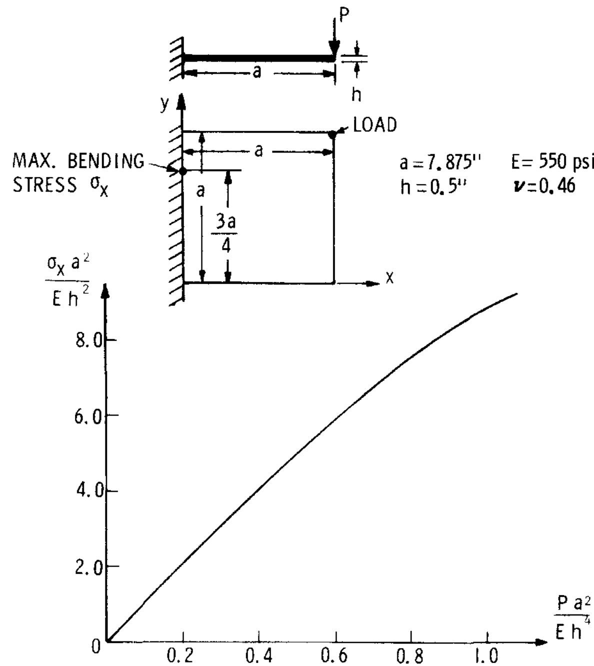

Fig. 11. Maximum bending stress σ_x along the top surface of the clamped edge in a square cantilever plate subjected to different load levels at the corner.

method for the same problem are included. In the same figure the results obtained from the strain gage test are also included. Non-dimensionalized bending stress (σ_x) along the top surface of the clamped edge and along the free edges AB and CD for different load levels are shown in Figs. 8, 9 and 10, respectively. Figure 11 shows the variation of the maximum bending stress (σ_x) along the top surface of the clamped edge as a function of the applied load. The results should be useful in the design of square cantilever plates subjected to a corner load and large deflections.

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REFERENCES

[1] F. K. Ligtenberg, The moire method—a new experimental method for the determination of moments in small slab models. *Proc. Soc. Exp. Stress Analysis* **XII**, 83–98 (1955).
[2] W. A. Bradley, Laterally loaded thin flat plates. *Proc. Am. Soc. civ. Engrs* **85**, 77–107 (1959).
[3] J. P. Duncan and C. J. E. Brown, Slope contours in flexed elastic plates by Salet–Ikeda technique. *Proc. 1st Int. Congr. Exp. Mech., Soc. Exp. Stress Analysis*, pp. 149–176 (1963).
[4] P. Dantu, Méthode nouvelle de détermination des contraintes en élasticité plane. *Ann. Ponts Chaussees* (1952).
[5] S. Morse, A. J. Durelli and C. A. Sciamarella, Geometry of moire fringes in strain analysis. *J. Engng Mech. Div. Am. Soc. civ. Engrs.* **86**, 105–126 (1960).
[6] C. A. Sciammarella and A. J. Durelli, Moire fringes as a means of analyzing strains. *J. Engng Mech. Div. Am. Soc. civ. Engrs* **87**, 55–74 (1961).
[7] A. J. Durelli and W. F. Riley, *Introduction to Photomechanics*, pp. 194, 246 and 359. Prentice-Hall (1965).
[8] A. J. Durelli, *Applied Stress Analysis*, pp. 60, 64, 78 and 169. Prentice-Hall (1967).
[9] V. J. Parks and A. J. Durelli, Various forms of the strain–displacement relations applied to experimental strain analysis. *Exp. Mech.* **4**, 37–46 (1964).
[10] S. Timoshenko and S. Woinowsky-Krieger, *Theory of Plates and Shells*, 2nd edition. McGraw-Hill (1959).
[11] T. H. Lin, S. R. Lin and B. Mazelsky, Large inextensional deflection of thin cantilevered plates. *J. appl. Mech.* **35**, 774–777 (1968).
[12] A. J. Durelli, V. J. Parks and L. Ferrer, Photography without lenses. *Exp. Mech.* **8**, 142–144 (1968).
[13] A. Ponce and H. Fernandez Tuneu, Estudio sistematico de placas en mensula por el metodo del moire. *Memorias de las IX Jornadas, Sud-Americanas de Ing. Estructural, Montevedieo.* Pub. No. 25, pp. 267–287 (1964).

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Абстракт—С целью анализа изгибаемых пластинок, описывается несложный экспериментальный метод, который обладает преимуществом в многих случаях, при отношении общепринятого определения наклонов с помощью метода муара из плоскости. Новый метод определяет перемещения в плоскости, путем использования метода муара в плоскости. Распределение деформаций и напряжений легко получается, используя растры муара, для пластинок подверженных действию так малым, как и большим прогибам. Для иллюстрации, приводятся два примера изгиба пластинок, один с малым прогибом, второй с большим. Они равняются относительно: свободно опертой квадратной пластинке, нагруженной равномерным давлением и квадратной консольной пластинке, подверженной действию нагрузке в угле.